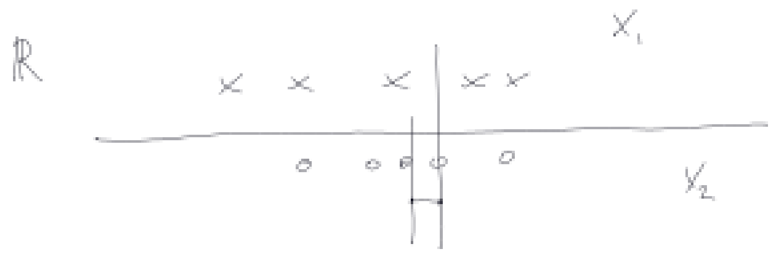
Decision Theory + NSA

admissibility = Bayes ~~+~~ infinitesimals



$X_{0,0}, \dots, X_{0,n} \overset{iid}{\sim} N(\mu, \sigma_0^2)$

$X_{1,0}, \dots, X_{1,n} \overset{iid}{\sim} N(\mu, \sigma_1^2)$

$\hat{\mu}_{\mathbb{D}}(X_{0,0}, \dots) = \bar{X}_0 + (\bar{X}_1 - \bar{X}_0) \frac{S_0^2}{S_0^2 + S_1^2}$

$S_i^2 = \frac{1}{n} \sum_j (\bar{X}_i - X_{i,j})^2$

$\bar{X}_i = \frac{1}{n+1} \sum_j X_{ij}$

Define
Decision probem : tuple (

$\mathcal{P} = (N_{(\mu,\sigma^2)})_{(\mu,\sigma^2) \in \Theta}$, $\Theta = \mathbb{R} \times \mathbb{R}_{>0}$

$\mathbb{X}$ ... sample space

P ... model $\mathcal{P} = \{ P_\theta : \theta \in \Theta \} \subseteq \mathcal{P}(\mathbb{X})$ $P_\theta(X \in B)$

Θ ... parameter space prior $\pi \in \mathcal{P}(\Theta)$

$\mathbb{A}$ ... actionspace A St observes x, takes action $a \in \mathbb{A}$, $a = \delta(x)$

ℓ ... $\ell : \Theta \times \mathbb{A} \to \mathbb{R}$ Goal might be to estimate $g(\theta)$, $\ell(\theta, a) = d(g(\theta), a)$

$\mathcal{D}$ ... randomized decision rules $\subseteq \{ \delta : \mathbb{X} \to \mathcal{P}(\mathbb{A}) \}$ $\mathcal{P}(\mathbb{A})$

EX

$\hat{\mu}_{\mathbb{D}}(x)$

$\hat{\mu} = \bar{x}$

$S^2 = \delta^2(x)$

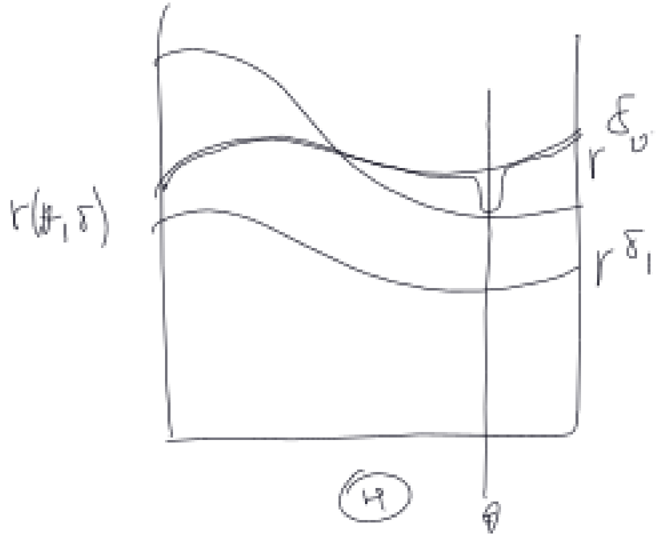
$\ell((\mu, \sigma^2), \hat{\mu}) = \|\mu - \hat{\mu}\|^2$

$$E\, \ell(\theta, \underbrace{\delta(X)}_{\in \mathbb{A}}) = \int \ell(\theta, a)\, \delta(x)(da)\, P_\theta(dx) =: r(\theta, \delta) = r^\delta(\theta)$$

$d\delta(x)(a)$

$\delta \rightsquigarrow r^\delta \in {}^{\Theta}\mathbb{R}$

$\boxed{}$ a^



among $\mathcal{C}\subseteq\mathcal{D}$

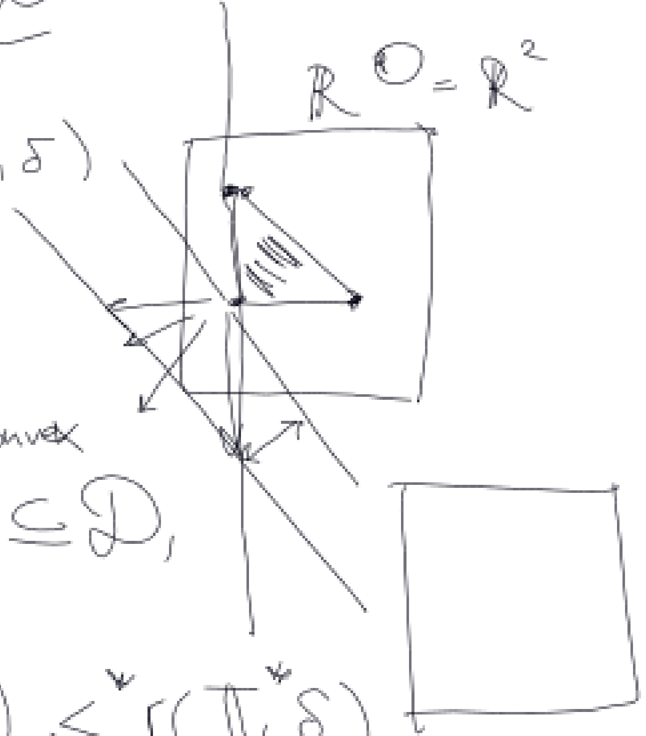
δ_0 admissible among iff $\neg(\exists\, \delta_1\in\mathcal{C} \quad \delta_1\prec\delta_0)$

$\delta_1 \prec \delta_0$ iff $\forall\theta \quad r^{\delta_1}(\theta)\le r^{\delta_0}(\theta)$

$\exists\theta \quad r^{\delta_1}(\theta)<r^{\delta_0}(\theta)$

δ_0 is Bayes under $\pi\in\mathcal{P}(\Theta)$ among $\mathcal{C}$

$$r(\pi,\delta_0)\le \inf_{\delta\in\mathcal{C}} r(\pi,\delta)$$



$$r(\pi,\delta)=\mathop{E}_{\theta\sim\pi} r(\theta,\delta)=\int r(\theta,\delta)\,d\pi(\theta)$$

<u>THM</u>: In an arbitrary decision problem, δ_0 is adm. among [extended] convex $\mathcal{C}\subseteq\mathcal{D}$,

iff $\exists\, \Pi \in {}^*(\mathcal{P}(\Theta))$ s.t. $\forall\delta\in\mathcal{D} \quad {}^*r(\Pi,{}^*\delta_0)\le {}^*r(\Pi,{}^*\delta)$

$[\forall\theta \quad \Pi(\{\theta\})>0]$

<u>THM</u>: If Θ is compact, δ_0 ext. adm. among $\mathcal{C}$ iff δ_0 is Bayes among $\mathcal{C}$